Stationarity *vs* Time Scale Dependence

or

Statistics *vs* Sample Path Variability

or

Tracking *vs* Estimation

MICRO-TUTORIAL

ISMA 2003 Bandwidth Estimation Workshop (BEst)

Darryl Veitch

“Estimating Available Bandwidth on Different Timescales”

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Sounds good –

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Must distinguish between:

- *Statistics* i.e. distributions, which are not random
  - – could be constant over time: *stationary*, or
  - – could be varying over time: *non-stationary*
  - – try to measure them: statistical estimation
- *Sample paths* these are ‘random functions’
  - – a single sample path is deterministic
  - – variability across and within paths is natural, regardless of statistics
  - – one path not enough for good estimation

Note: Using conceptual random process **model** – unavoidable, and natural

A Random Variable: Distribution, Samples and Estimation

Consider a continuous r.v. X, with

- Distribution function (CDF) $F(x) = \mathrm{Pr}(X \leq x)$, probability density $f(x) = F'(x)$,
- *Expectation*: $\mu_X = \mathbf{E}[X] = \int x f(x)\, dx$, *Variance*: $\sigma_X^2 = \mathbf{E}[(X - \mu)^2]$,

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Now consider a sample value x of X. How to estimate μ_X?

Could set $\widehat{\mu} = x$. Really this is a sample value of an *Estimator*

- *Estimator*: $\widehat{\mu}_X = X$ is a r.v.
- With one sample x of X, sample of $\widehat{\mu}$ is also x.
- Not a great solution since although $\mathbf{E}[\widehat{\mu}] = \mu$, have $\text{Var}\,[\widehat{\mu}] = \sigma^2$.
- Can't even estimate σ_X^2 with this, or much else!

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Would like to have more (independent) samples available:

Could set $\widehat{\mu}_n = \bar{X} \equiv (\sum_i^n X_i)/n$, with sample values x_i, $i = 1, 2 \cdots$.

- $\mathbf{E}[\widehat{\mu}] = \mu$, $\text{Var}\,[\widehat{\mu}] = \sigma^2/n$.
- Gets rapidly better with increasing n!
- Now could estimate σ^2, e.g. using $S^2 = \sum_i^n (X_i - \bar{X})^2/(n-1)$, the 'sample variance' (another r.v.).

A Random Time Series: Distributions, Samples and Estimation

Consider a time series $X(t)$, say $t \in \mathcal{Z}$.

- Distribution functions $F_{X(t)}(x)$ for all t, and 2-D joint $F_{X(t),X(t')}(x, x')$, all 3-D ....
- Expectations $\mu_{X(t)}$, variance, 3rd order moments, etc for each t.
- All statistics formed from joint distributions from any combination of the $X(t)$'s.

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Now consider a sample path $x(t)$. How to estimate $\mu_5 = E[X(5)]$?

As $X(5)$ is just one r.v., as before could set $\widehat{\mu}_5 = X(5)$.

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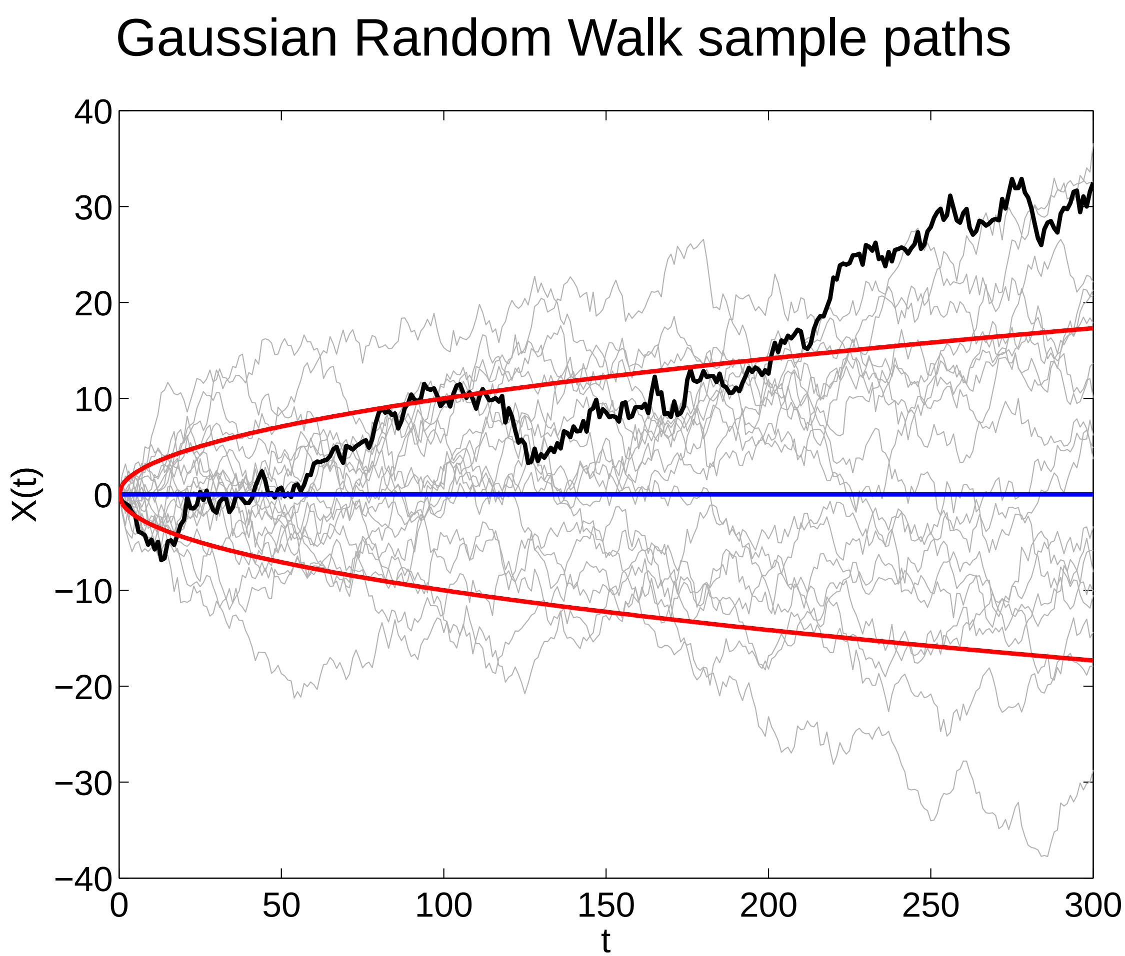
Would like to have more (independent) samples available:

But there aren't any! unless:

- We assume stationarity (and ergodicity), or
- We use simulation, or
- We assume real data can 'repeat itself', and can affort to wait.

Estimating With Non-Stationarity - how hard it can be



Estimates on 'different time-scales'

- Not only time-scale, but time instant matters.
- Time varying statistics and sample variability mixed together.
- Only get 'one sample' of everything.

The Stationary Case

Consider a stationary time series $X(t)$.

- Stationary means that all statistics are time-origin invariant:
- eg.1: Marginal distributions all the same: $F_{X(t)} = F_{X(t')}$ (so $\mathbf{E}[X(t)] = \mathbf{E}[X(t')]$ etc.).
- eg.2: *Covariance Function*: $\gamma(t - t') = \mathbf{E}[(X(t) - \mu)(X(t') - \mu)]$ depends only on $t - t'$
- eg.3: Any statistic formed from any combination of the $X(t)$'s.

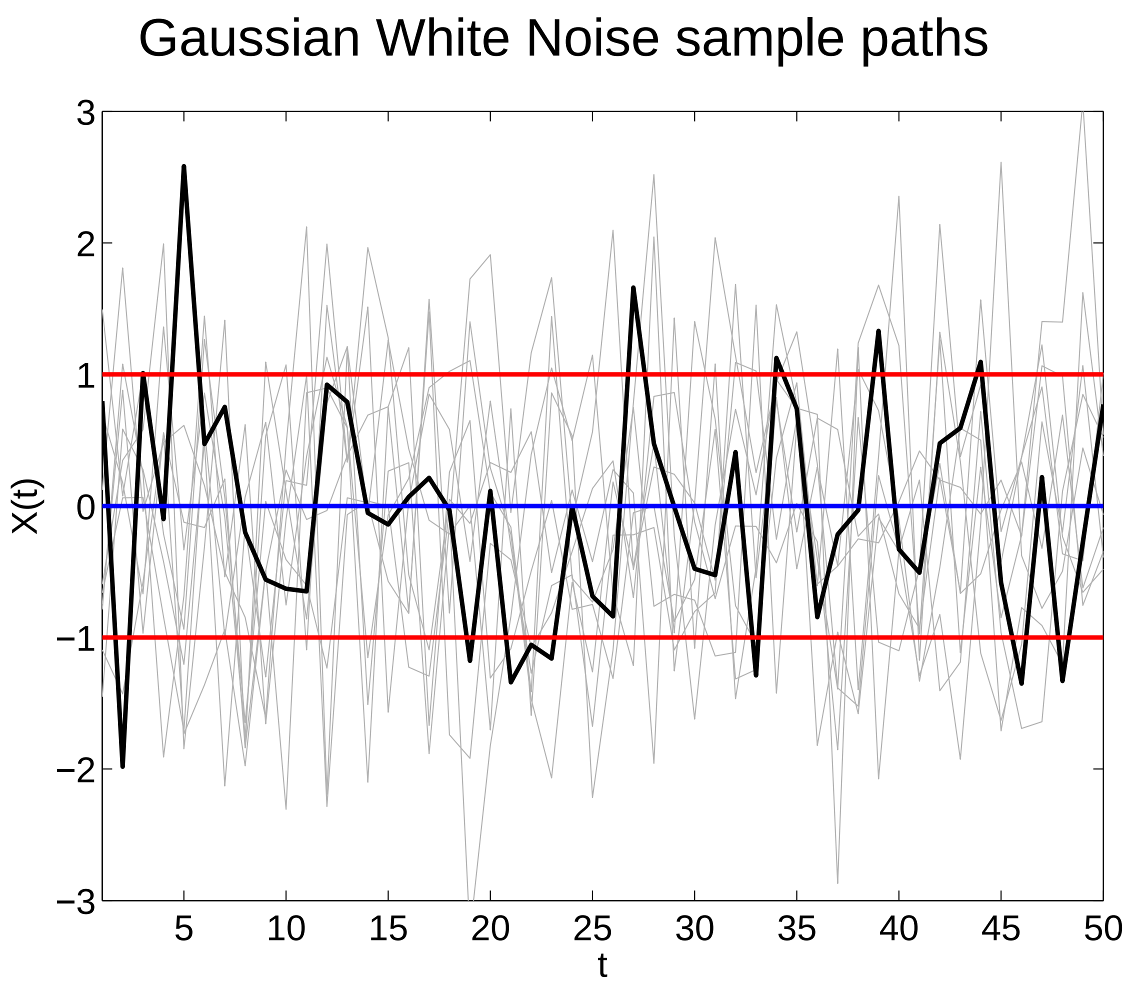
Now consider a sample path $x(t)$. How to estimate $\mu = \mu_X(t)$?

Set $\widehat{\mu} = \bar{X}$ as before:

- $\mathbf{E}[\widehat{\mu}] = \mu$, Var $[\widehat{\mu}] = \sigma^2/n$ (in white noise case)
- Now gets better with n.

More samples are available, and statistics are constant

Estimating With Stationarity - how hard can it be?

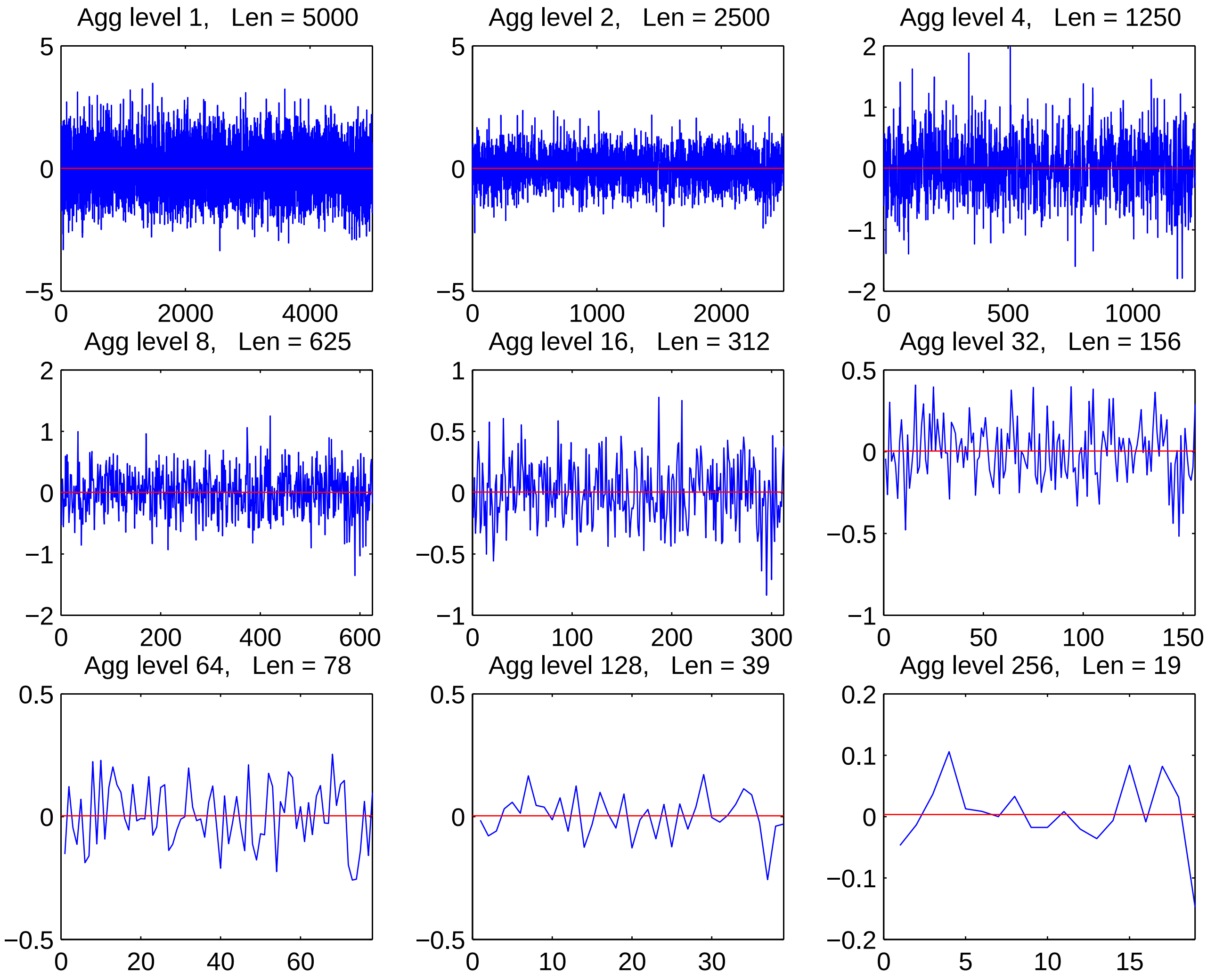


Estimates on ‘different time-scales’

- Nothing varies with time! – only have to worry about sample variability.
- Effectively get many samples of everything.
- Different time scales relates to estimation variance.

Time Scale Dependence of Estimates

Gaussian White Noise aggregated over different time intervals



Estimating Available Bandwidth on Different Timescales

AB will change with time – we want to track it.

However, can't estimate an arbitrary non-stationarity, so:

- Must assume stationarity over some time interval T
- With T bounded, so is estimation quality

Measurement over Different timescales refers to the tradeoff :

- Small T makes stationarity assumption better
- But estimation variance (assuming stationarity) worse