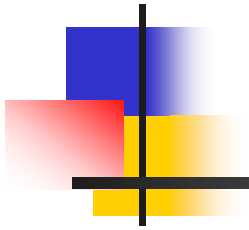

Detection of Topological Patterns in Complex Networks: Correlation Profile of the Internet

Sergei Maslov

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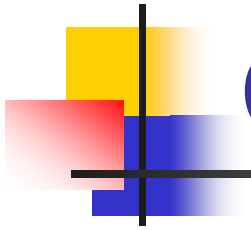
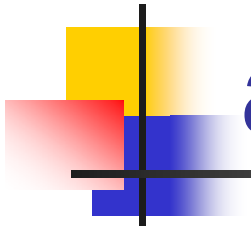


Outline

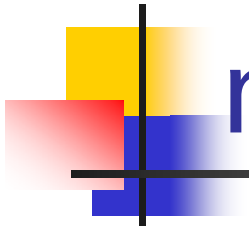
- Two intuitive algorithms to construct a randomized network with a given degree distribution
  - S. Maslov, K. Sneppen, Science (2002)
  - S. Maslov, K. Sneppen: cond-mat/0205379 at arxiv.org (2002), Physica A (2004)
- Apply them to detect the 3D plot of degree-degree correlations in the Internet
- Is Internet really disassortative?

Which topological patterns are important?



- Which topological patterns of a large complex network are there for a reason:
  - design principles, functional constraints
  - generated by growth dynamics
- Compare the number of patterns in real and properly randomized (null model) networks

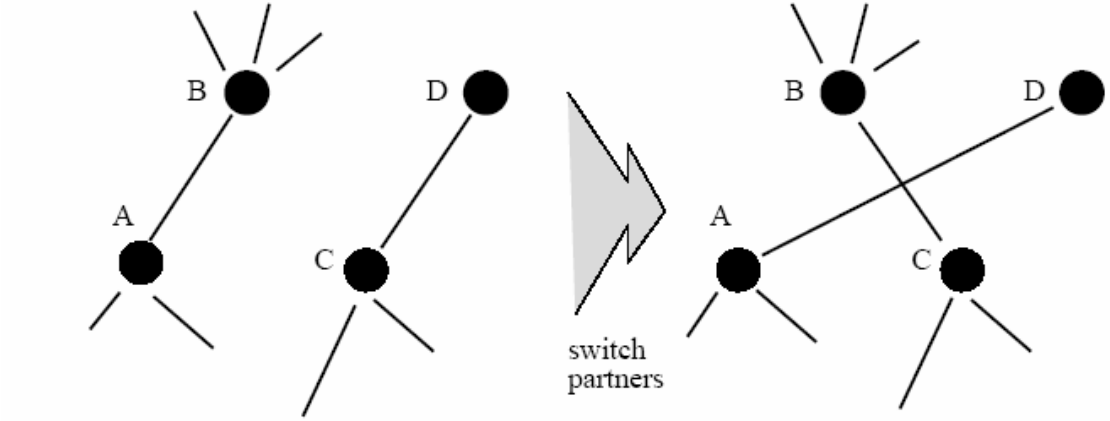
What to include in the null model?



- Measurable quantities that you deem important!
  - Degrees of individual nodes
  - Global connectivity
  - Clustering, geography, user-provider status, etc.
- To discover novel high-level patterns the null model should include all low-level patterns that are “understood” (or commonly accepted)

How to construct a proper random network?

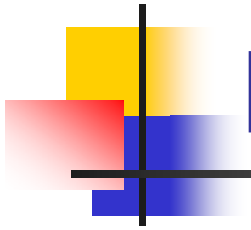
The basic edge swapping (rewiring) algorithm



- Randomly select and rewire two edges
- Repeat many times

S. Maslov,
K. Sneppen,
Science (2002)

R. Kannan,
P. Tetali,
S. Vempala,
Random Structures
and Algorithms (1999).



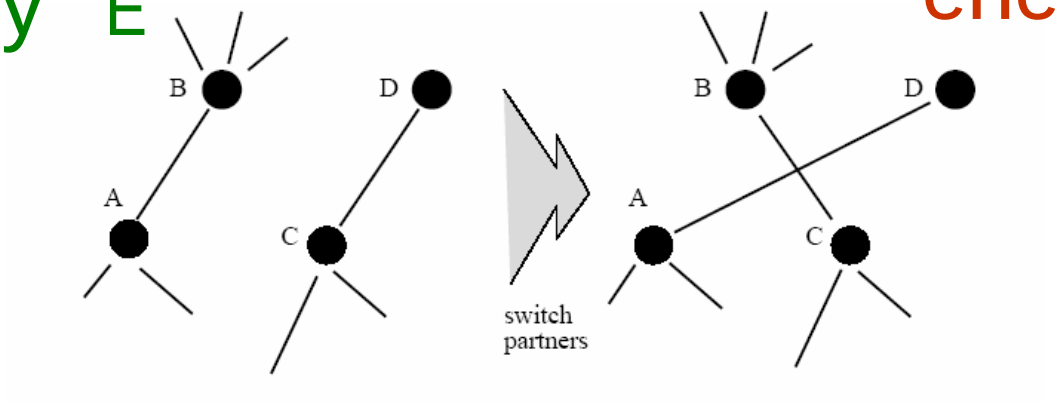
No multiple edges

- When constructing a random network – do not allow multiple edges
- Expected number of edges between a pair of nodes is $E_{ij}=K_iK_j/(2E)$
- E_{h1h2} between the two largest hubs in the Internet circa January 2000 is $1458 * 750/ (2 * 12{,}573)=43.5$ edges!
- Dangerous for $\gamma<3$ as [# of hub-to-hub edges] $\sim N^{(3-\gamma)/(\gamma-1)}$

Rewiring algorithm with a twist

"energy" E

"energy" E+ΔE



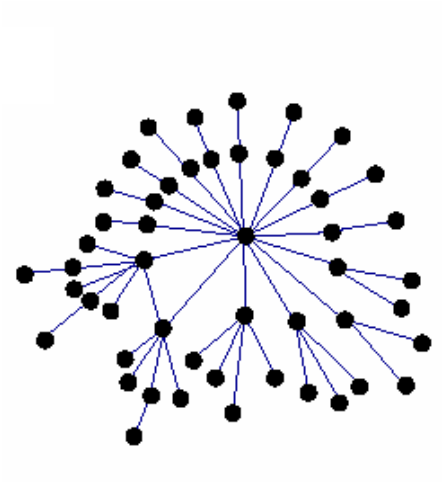
- Define energy function $E=(N_{actual}-N_{desired})^2/N_{desired}$
  - N_{actual} - the actual number of e.g. triangles
  - $N_{desired}$ – what we want it to be
- Randomly select two edges and calculate change ΔE in the energy function
- Rewire with probability $p=\exp(-\Delta E/T)$

S. Maslov, K. Sneppen: cond-mat preprint at arxiv.org (2002)

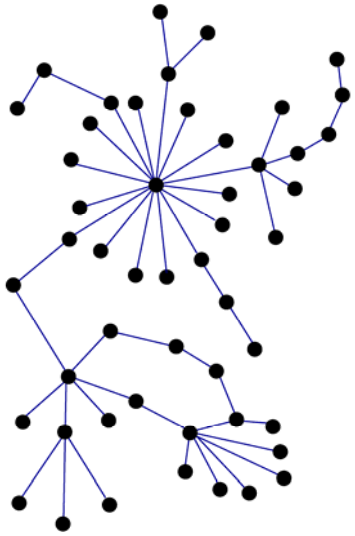
Published with A. Zaliznyak Physica A (2004)

Beyond degree distributions: How is it all wired together?

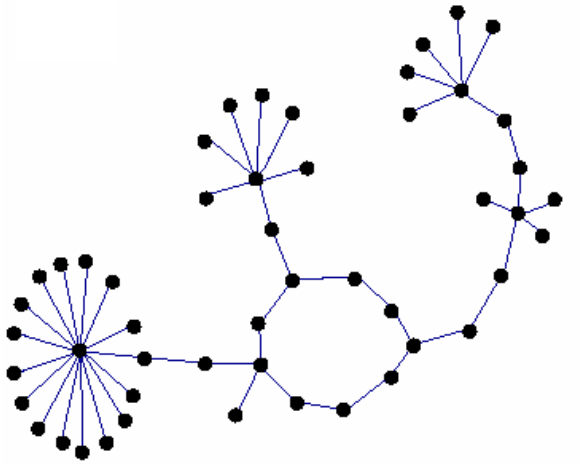
Central vs peripheral network architecture



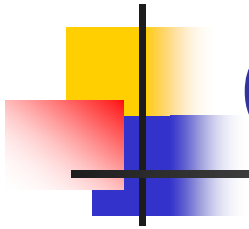
Largest hub is in the center (very hierarchical) “assortative”



Random



Hubs are peripheral (very anti-hierarchical) “disassortative”

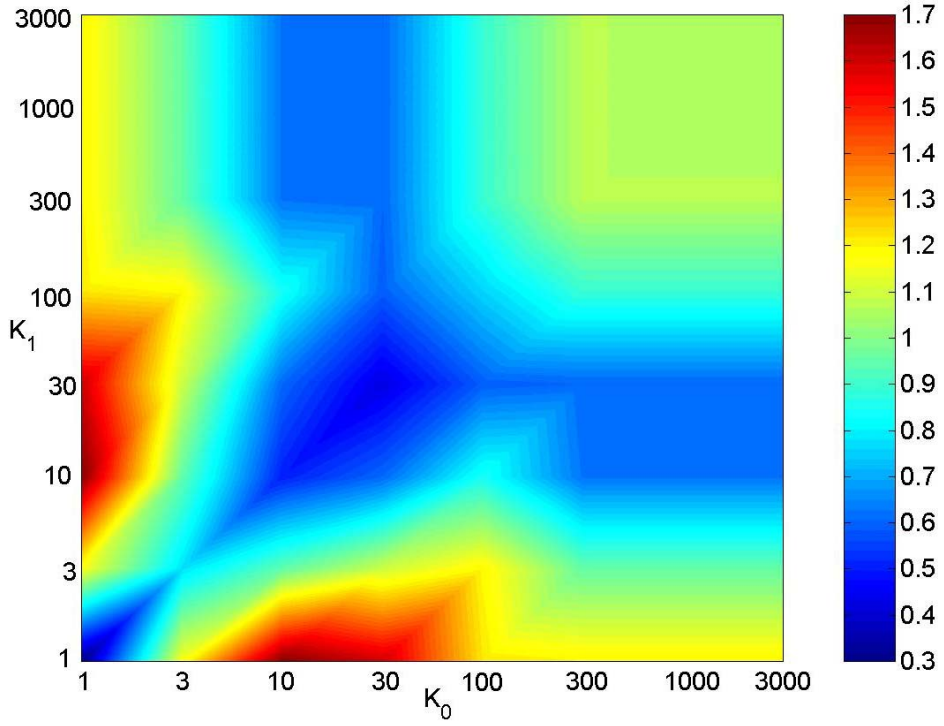


Correlation profile

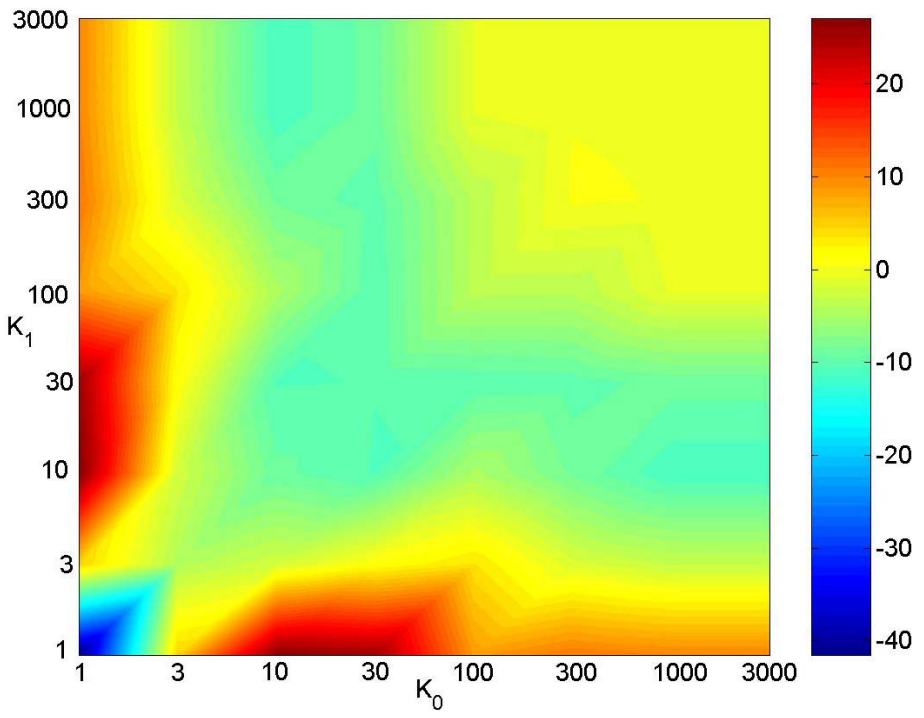
- Count $N(k_0,k_1)$ – the number of links between nodes with connectivities k_0 and k_1
- Compare it to $N_r(k_0,k_1)$ – the same property in a randomized network
- Randomized network conserves degrees of individual AS and the single-component nature of the Internet

Degree-degree correlations in the AS-network

$N(k_0,k_1)/ N_r(k_0,k_1)$

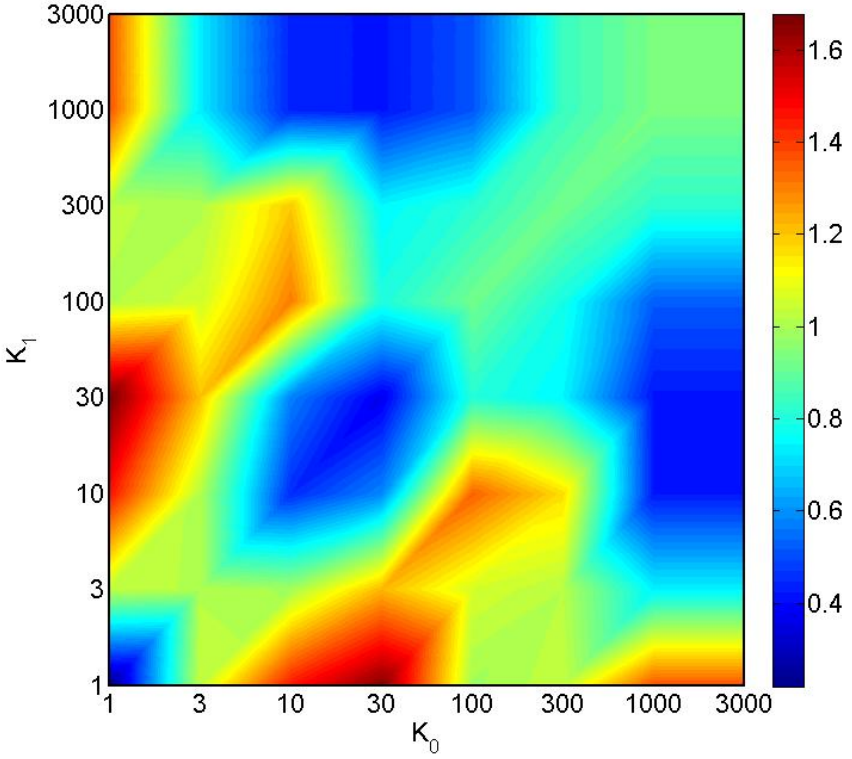


$[N(k_0,k_1)-N_r(k_0,k_1)] / \Delta N_r(k_0,k_1)$

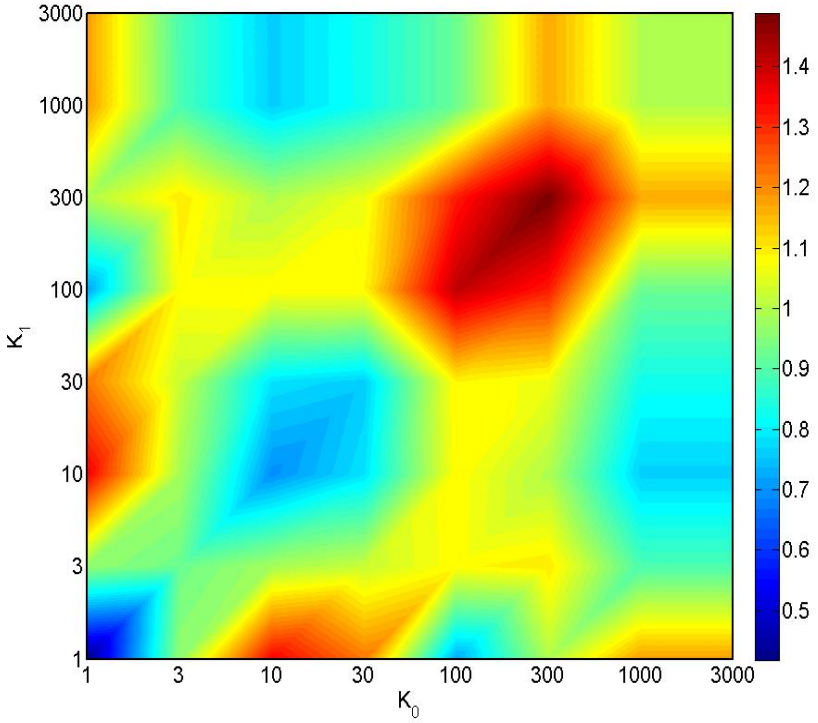


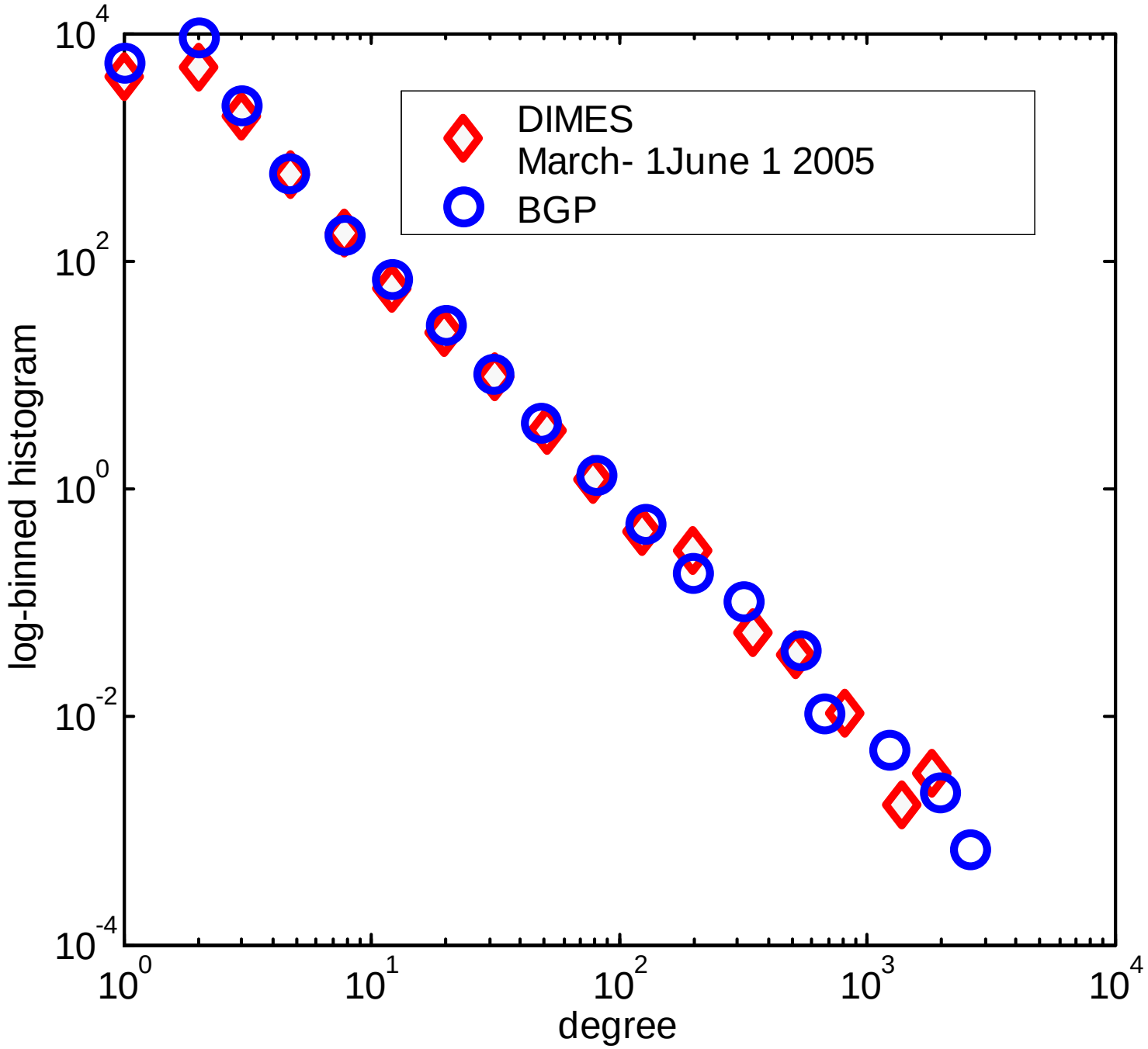
Does it hold for recent data?

BGP, March 1-June 1 2005

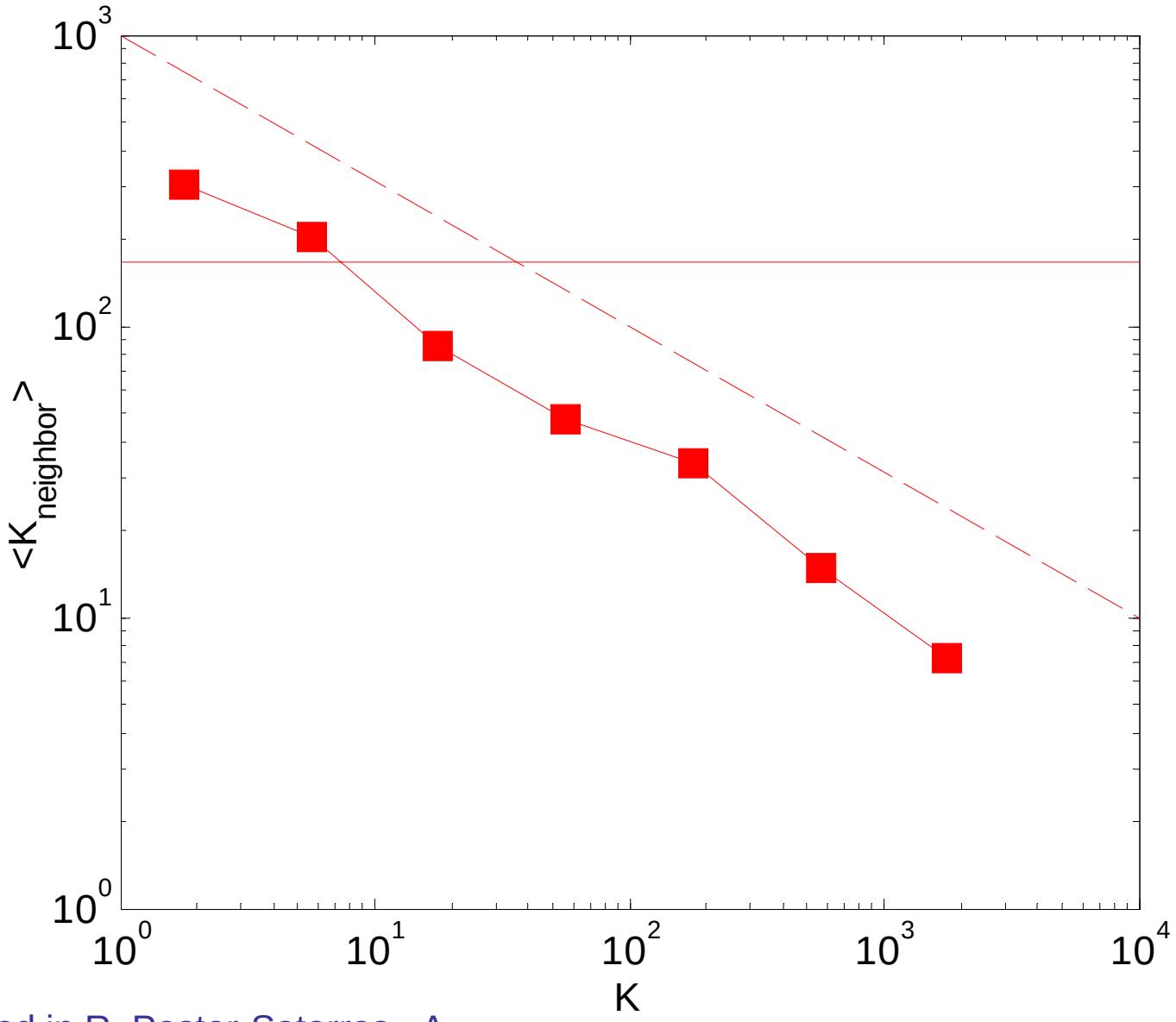


DIMES, March 1-June 1 2005

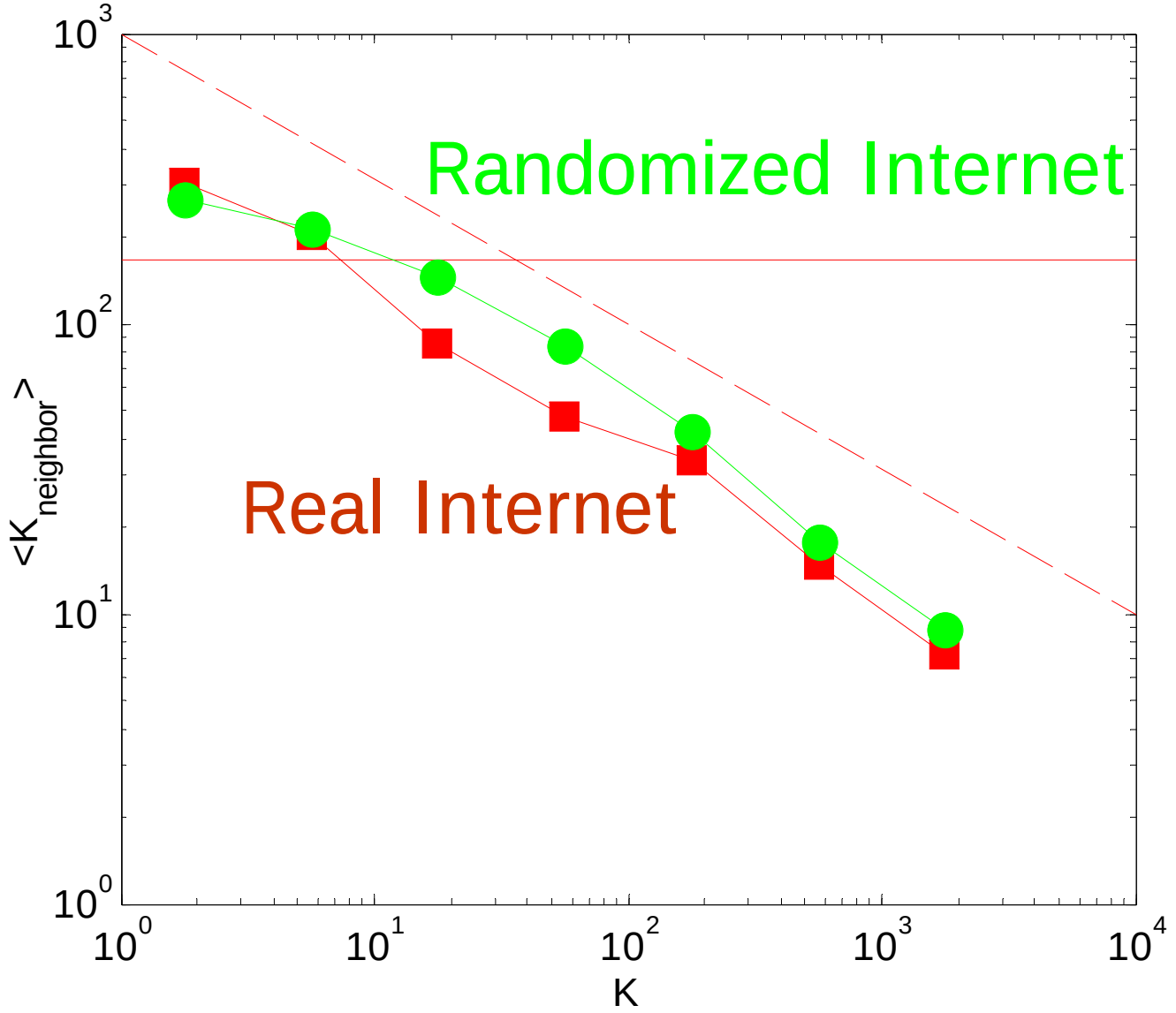



DIMES
March- 1June 1 2005
BGP
log-binned histogram
degree
10^4
10^2
10^0
10^-2
10^-4
10^0
10^1
10^2
10^3
10^4

Is Internet disassortative?

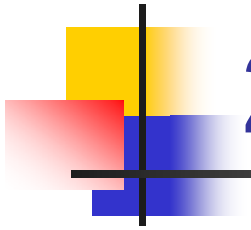


First reported in R. Pastor-Satorras, A. Vázquez,
and A. Vespignani, Phys. Rev. Lett. (2001)



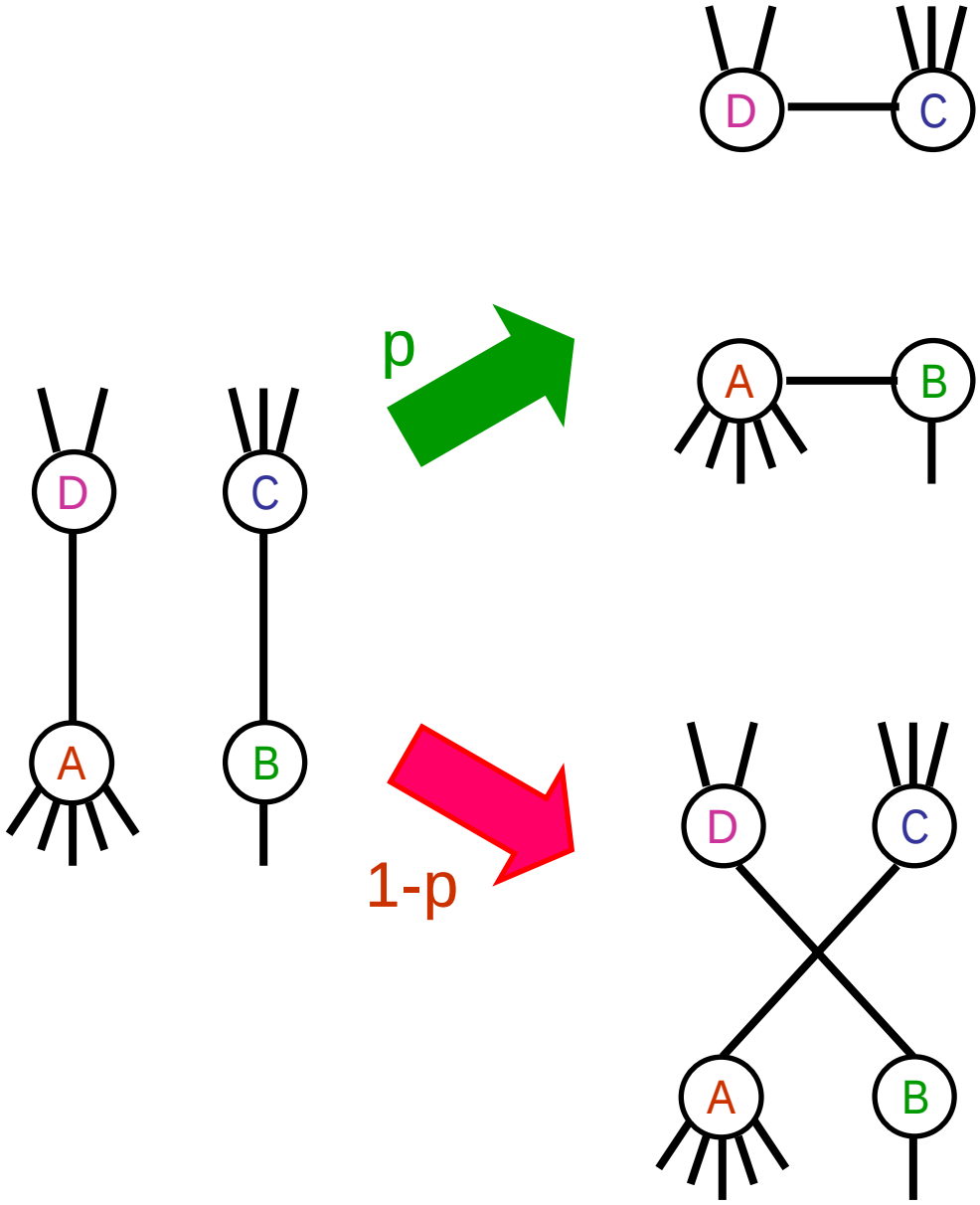
S. Maslov, K. Sneppen,
cond-mat/0205379, (2002)

What to include in a proper random network?



2nd generation of null models

- N(k,k') may be conserved in addition to N(k)
- The null model could be generated by our rewiring algorithm with energy function
  - Bin the connectivity k into few bins per decade
- For a crude model one could use our hierarchical/anti-hierarchical rewiring model
  - A. Trusina, S. Maslov, P. Minnhagen, and K. Sneppen, Phys. Rev. Lett. 92, 17870 (2004), cond-mat/0308339.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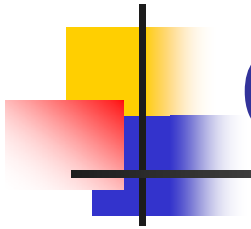

Conclusions

- Internet is NOT disassortative!
- Network rewiring with a twist – a useful tool to generate random networks with desired low-level topological properties
- Could be used to discover non-random topological features e.g. degree-degree correlations (and much more)
  - Super-hubs do not avoid other super-hubs in the AS-Internet (an artifact of multiple edges in a null model)
  - Mid-sized nodes like to connect to “user” nodes (degrees 1-3)
  - User nodes avoid other user nodes

THE END